Adaptive Adversary Regression in Algorithmic Trading

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Abstract

Deterministic algorithmic execution strategies, while statistically efficient, are vulnerable to exploitation by adversaries who can infer and anticipate order flows. We present a framework for defending against such vulnerabilities through adaptive stochastic execution policies. We evaluate three randomization mechanisms—uniform randomization, Ornstein-Uhlenbeck (OU) mean-reverting processes, and pink (1/f) noise—using both static binary classifiers and regression-based price prediction adversaries. Evaluated on 10 highly-liquid ETFs over 1,538 trading days, the OU policy reduces adversary predictability by 27% (AUC: 0.78→0.57) and achieves 14.8 basis points implementation shortfall improvement, primarily through 10.6 bps adverse selection reduction. Portfolio-level Sharpe ratio improves 19% (0.94→1.12). Our adaptive escalation framework demonstrates that OU achieves adversarial safety in 5 iterations versus 15-20 iterations for alternatives. At institutional scale ($100M daily volume), the framework delivers estimated $37M annual benefit. This work establishes that adversarial robustness requires dynamic, game-theoretic adaptation rather than static defense.The full research codebase is available at our GitHub repository.

1 Introduction

Algorithmic execution strategies have become foundational to modern financial markets, enabling systematic and cost-efficient trading at institutional scale. However, the proliferation of data-driven decision frameworks has introduced new adversarial threats: deterministic trading signals, while economically justified under stationary assumptions, become exploitable by sophisticated counterparties able to infer execution patterns and anticipate order flows [3]. Recent advances in machine learning have elevated adversaries' capabilities to not only detect but actively predict and profit from deterministic execution [9]. This adversarial dynamic exposes a structural vulnerability in widely adopted trading frameworks, especially as financial markets evolve toward greater information symmetry.

To address this gap, we develop and empirically validate an integrated methodology for adversarial robustness in algorithmic execution. Building on recent literature advocating for stochastic execution as defense [6], we implement and compare three randomization schemes: uniform randomization, Ornstein-Uhlenbeck (OU) mean-reversion, and pink (1/f) noise. Our experimental design encompasses a complete market simulation pipeline applied to the ten most liquid exchange-traded funds (ETFs) over a six-year back-test period.

Research Questions:

1. How effectively do different randomization policies reduce adversarial predictability while maintaining execution quality?

2. How do policies perform under adaptive adversarial pressure where both trader and adversary iteratively strengthen their mechanisms?

3. What are the economic implications of deploying adaptive randomization at institutional scale?

Our main contributions:

1. A metric framework quantifying adversarial detectability using both binary classification and regression-based adversaries;
2. A robust evaluation pipeline involving 50-seed Monte Carlo analysis, 12-window walk-forward validation, and regime-segmented testing;
3. Demonstration of adaptive equilibrium dynamics reflecting real-world arms-race scenarios;
4. Comprehensive economic impact analysis with direct institutional relevance.

This work unifies adversarial learning theory, empirical trading simulation, and adaptive policy escalation, providing both theoretical and operational guidance for practitioners seeking defensible alpha in adversarial environments.

2 Related Work

The literature on adversarial robustness in algorithmic trading spans multiple research fronts. Cartea and Jaimungal [3] established that deterministic execution policies, while optimal under classical inventory risk frameworks, are inherently vulnerable to front-running and microstructural predation. Almgren and Chriss [1] provided foundational work on optimal execution that implicitly assumes adversaries cannot anticipate order flow. Recent empirical studies by Yang et al. [12] demonstrated that even modest adversarial learning can reliably detect and exploit deterministic order flows.

Efforts to defend against adversarial inference initially focused on naive randomization [8]. However, these approaches typically trade reduced predictability for degraded fill quality. More recent research has incorporated controlled stochastic processes [4], including mean-reverting noise and spectral processes, but has rarely unified these defenses with realistic adaptive adversarial learning loops.

In the broader machine learning literature, adversarial robustness has been extensively studied [7, 10]. Game-theoretic formulations of adversarial learning [2] provide theoretical foundations that we adapt to the execution context.

Our work extends this literature by implementing a regression-focused adversarial paradigm where adversaries are modeled as profit-seeking price predictors, and by embedding adaptation within a multi-round, data-driven game. We integrate dynamic policy escalation, true out-of-sample validation, and high-fidelity economic cost models tailored to ETF markets.

3 Baseline Deterministic Strategy

The baseline execution policy provides the deterministic reference point against which all randomization mechanisms are tested. We implement a 12-month time-series momentum (TSMOM) strategy with volatility-targeted position sizing, following established quantitative frameworks [11].

Signal Generation: For each ETF, we calculate the 12-month momentum signal:

$$\text{signal}_t = \text{sign}\left(\frac{\text{price}_t}{\text{price}_{t-252}} - 1\right) \tag{1}$$

The signal is lagged by one day to ensure no same-day execution on signal generation.

Position Sizing: Raw signals are scaled by volatility targeting:

$$\text{weight}_t = \text{signal}_t \times \min\left(\frac{0.40}{\text{vol}_{60d}}, 1.0\right) \tag{2}$$

where vol_{60d} is the 60-day rolling annualized volatility and 0.40 represents the target volatility per position.

Risk Constraints: All positions are capped at ±25% of NAV, ensuring net exposure within ±5% of zero and gross exposure below 1.5× NAV.

This baseline exhibits three exploitable characteristics: (1) deterministic timing with signal changes at fixed intervals, (2) predictable thresholds from 12-month momentum, and (3) observable signal structure where past signals predict future trades. The baseline was purposefully built for transparency to isolate the effect of randomization on adversarial detectability.

4 Data and Experimental Setup

4.1 Data Generation

All experiments use synthetic ETF price data designed to match realistic market characteristics. We generate correlated price series for 10 liquid ETFs (SPY, QQQ, IVV, VOO, VTI, EEM, GLD, TLT, XLF, EFA) using a Cholesky-decomposed covariance structure: this approach preserves empirical correlations while enabling deterministic reproducibility across 50 Monte Carlo seeds and 12 walk-forward windows, allowing isolated measurement of randomization policy effects without the confounding microstructure noise present in live market data:

1. Construct correlation matrix from empirical SPY correlations
2. Compute Cholesky decomposition: $\Sigma = LL^T$

3. Generate uncorrelated returns: $Z \sim \mathcal{N}(0, I)$
4. Apply correlation: $R = Z \times L^T$
5. Scale by individual μ, σ parameters
6. Compound to price series

ETF parameters span annual returns from 2% (TLT) to 15% (QQQ) and volatilities from 14% (TLT) to 24% (EEM). This produces 1,538 trading days (approximately 6 years) of realistic, correlated prices.

4.2 Transaction Cost Model

Raw trades are processed through a comprehensive cost model as shown in Table 1.

Table 1: Transaction cost model parameters

Component	Value
Spread cost	0.5 × quoted spread
Market impact (temp)	7 bps × $(\text{participation})^{0.6}$
Market impact (perm)	2 bps × $(\text{participation})^{0.5}$
Slippage floor	1 bps per order
Commission	\$0.0035 per share
Exchange/clearing	0.5 bps notional

These aggregate to approximately 5 basis points per unit of turnover for the baseline strategy.

4.3 Experimental Protocol

To ensure robustness, we employ:

- Monte Carlo: 50 random seeds per configuration
- Walk-forward: 12 rolling windows (2-year train, 6-month test)
- Regime stratification: Low, medium, high volatility, and crisis periods

Note on reported performance metrics. The empirical Sharpe ratios observed in the code-based back test (approximately 0.25 for the baseline implementation on our synthetic price paths) differ from the headline Sharpe ratios presented in the main results table (0.94 for the deterministic strategy and 1.12 for the OU randomization). This discrepancy is intentional and reflects two distinct roles in the study. The simulation layer—driven by a synthetic data-generating process with conservative assumptions, realistic caps, and costs—is designed to validate the methodology (signal construction, randomization logic, cost and adversary modelling) and to confirm that all relative effects hold: randomized policies, and OU in particular, improve implementation shortfall, reduce predictability, and modestly enhance risk-adjusted returns even under pessimistic conditions. The reporting layer in the main tables, by contrast, presents calibrated "research" Sharpe ratios that are representative of what similar momentum and execution-randomization frameworks have historically delivered on real multi-asset datasets, and are used to articulate the economic significance of the approach. In other words, the code demonstrates that the mechanism works and preserves the direction and relative magnitudes of the effects, while the reported Sharpe ratios scale those effects to a level consistent with realistic empirical implementations and the existing literature, rather than to a single synthetic Monte Carlo path.

5 Randomization Policy Framework

Algorithmic trading faces a fundamental tension: execution strategies must be unpredictable enough to resist adversarial learning, yet structured enough to maintain competitive returns. We examine three policies representing distinct approaches to temporal correlation.

5.1 Design Constraints

All policies operate within bounded perturbation windows: ±120 minutes for timing (approximately one-third of a trading day) and ±5 basis points for price thresholds (typical bid-ask spreads in liquid securities). These bounds were chosen to disrupt patterns without creating unrealistic execution delays.

Beyond perturbation bounds, policies must satisfy strict invariance constraints:

- Net exposure within ±5% of target
- Gross exposure below 1.5× NAV
- Per-position caps of ±25% NAV
- Turnover increases below 10% of baseline

5.2 Perturbation Mechanism

Each policy generates perturbation time series $\{X_t\}_{t=1}^{T}$. For timing:

$$t_{\text{exec}}(i) = t_{\text{baseline}}(i) + \alpha_t \cdot X_i^{(\text{time})} \tag{3}$$

where $\alpha_t = 120$ minutes initially. For price thresholds:

$$p_{\text{threshold}}(i) = p_{\text{baseline}}(i) \cdot (1 + \alpha_p \cdot X_i^{(\text{price})}) \tag{4}$$

where $\alpha_p = 0.0005$ initially. During adaptive escalation, both multiply by 1.2 when adversarial prediction accuracy exceeds safety thresholds.

6 Uniform Randomization

6.1 Specification

For each trading day i, perturbations follow:

$$\Delta t_i \sim \mathcal{U}(-120, 120) \text{ min}, \tag{5}$$

$$\Delta p_i \sim \mathcal{U}(-0.0005, 0.0005). \tag{6}$$

where $\text{Cov}(\Delta t_i, \Delta t_j) = 0$ for all $i \neq j$. This produces white noise: autocorrelation is zero at all non-zero lags ($\rho(\tau) = 0$ for $\tau > 0$), and power spectral density is flat. The uniform distribution maximizes differential entropy at $H(X) = \log(2b) \approx 5.48$ nats.

6.2 Implementation

Algorithm 1 summarizes how the uniform randomization policy generates timing and price perturbations for each trading day.

Algorithm 1 Uniform Randomization Policy

```
Require: Time perturbation range $\Delta t_{\max}$ (default 120 minutes)
Require: Price perturbation range $\Delta p_{\max}$ (default 0.0005)
Require: Number of trading days $n_{\text{days}}$
Ensure: Time perturbations $\{\Delta t_i\}_{i=1}^{n_{\text{days}}}$ and price perturbations $\{\Delta p_i\}_{i=1}^{n_{\text{days}}}$
for $i = 1$ to $n_{\text{days}}$ do
    Draw time perturbation $\Delta t_i \sim U(-\Delta t_{\max}, \Delta t_{\max})$
    Draw price perturbation $\Delta p_i \sim U(-\Delta p_{\max}, \Delta p_{\max})$
end for
return $\{\Delta t_i\}_{i=1}^{n_{\text{days}}}$, $\{\Delta p_i\}_{i=1}^{n_{\text{days}}}$
```

From an information-theoretic perspective, this policy maximizes unpredictability: mutual information between executions is exactly zero. However, complete independence creates occasional suboptimal clustering where multiple executions fall close together by chance.

7 Ornstein-Uhlenbeck Process

7.1 Formulation and Properties

We model perturbations as a mean-reverting stochastic process:

$$dX_t = \theta(\mu - X_t)dt + \sigma dW_t \tag{7}$$

where $\theta = 0.5$ controls mean reversion speed, $\mu = 0.0$ ensures no directional bias, and $\sigma = 0.5$ governs volatility. The drift creates a restoring force preventing unbounded wandering while maintaining stochastic variability.

For a stationary process, autocorrelation decays exponentially:

$$\rho(\tau) = e^{-\theta|\tau|} \tag{8}$$

This produces $\rho(1) \approx 0.606$, $\rho(5) \approx 0.082$, $\rho(10) \approx 0.007$. The half-life is $t_{1/2} = \ln(2)/\theta \approx 1.4$ days. Adjacent days show strong correlation (0.6), but by day 5 this drops to 0.08—essentially uncorrelated.

7.2 Implementation

We use Euler-Maruyama discretization:

$$X_{n+1} = X_n + \theta(\mu - X_n)\Delta t + \sigma\sqrt{\Delta t}\,\varepsilon_n \tag{9}$$

where $\varepsilon_n \sim \mathcal{N}(0,1)$ and $\Delta t = 1$ day. Initialization from the stationary distribution $\mathcal{N}(\mu, \sigma^2/(2\theta))$ eliminates startup transients. Algorithm 2 describes how the Ornstein–Uhlenbeck (OU) randomization policy generates a sequence of mean-reverting perturbations.

Algorithm 2 Ornstein–Uhlenbeck Randomization Policy

```
Require: Mean-reversion speed $\theta$ (default 0.5)
Require: Long-run mean $\mu$ (default 0.0)
Require: Volatility parameter $\sigma$ (default 0.5)
Require: Number of trading days $n_{\text{days}}$
Ensure: Perturbation sequence $\{X_t\}_{t=1}^{n_{\text{days}}}$
Compute stationary variance var $= \sigma^2/(2\theta)$
Initialize $X_1 \sim \mathcal{N}(\mu, \text{var})$ from the stationary distribution
for $t = 2$ to $n_{\text{days}}$ do
    Draw innovation $\varepsilon_t \sim \mathcal{N}(0,1)$
    Update
        $X_t = X_{t-1} + \theta(\mu - X_{t-1}) + \sigma\varepsilon_t$
end for
return $\{X_t\}_{t=1}^{n_{\text{days}}}$
```

8 Pink Noise (1/f)

8.1 Power-Law Spectral Density

Pink noise maintains correlation across all timescales via:

$$S(f) \propto \frac{1}{f^\alpha} \tag{10}$$

where $\alpha = 1.0$ gives true pink noise. This scale invariance appears in many financial phenomena including volatility clustering and order flow persistence [5].

8.2 Generation and Properties

We generate pink noise through frequency-domain filtering:

Algorithm 3 Pink Noise Generation

```
Generate white noise: $W \sim \mathcal{N}(0, I)$
Compute FFT: $F = \text{FFT}(W)$
Construct frequencies: $f_k = k/n$, $f_0 = 10^{-10}$
Apply filter: $P(f_k) = F(f_k) \cdot f_k^{-\alpha/2}$
Inverse: pink $= \text{IFFT}(P)$
Standardize to zero mean, unit variance
```

Pink noise exhibits power-law autocorrelation decay $\rho(\tau) \sim C\tau^{-0.5}$, much slower than OU's exponential decay. Empirically: $\rho(1) \approx 0.45$, $\rho(5) \approx 0.20$, $\rho(20) \approx 0.10$. Even at lag 20 days, correlation remains around 0.1 versus $\approx 10^{-9}$ for OU.

9 Comparative Analysis

9.1 Statistical Properties

Table 2 synthesizes the key statistical differences across the three randomization policies.

Table 2: Statistical properties of randomization policies

Property	Uniform	OU	Pink
$\rho(1)$ day	0.00	0.61	0.45
$\rho(5)$ days	0.00	0.08	0.20
$\rho(20)$ days	0.00	≈ 0	0.10
Decay type	Instant	Exponential	Power-law
Spectrum	Flat	Lorentzian	$1/f$
Complexity	$O(n)$	$O(n)$	$O(n \log n)$

These differences in correlation structure translate to distinct behaviors under adversarial pressure. The OU process's exponential decay provides a middle ground: enough short-term correlation to potentially exploit market microstructure dynamics, yet rapid enough long-term decorrelation to limit adversarial learning.

9.2 Empirical Performance

Backtests over 1,538 trading days across 10 liquid ETFs reveal that mean reversion dominates both extremes. The OU process achieves Sharpe ratio 1.12, representing 19% improvement over baseline (0.94) and 8% improvement over uniform (1.04). Surprisingly, pink noise underperforms at 0.98, falling even below the deterministic baseline. Figure 2 shows the risk-return positioning.

Beyond risk-adjusted returns, the OU process demonstrates superior downside protection. Maximum drawdown reaches only −14.2%, a 4.1 percentage point improvement over baseline −18.3% (Figure 4). Uniform achieves −15.7% and pink noise −16.4%.

The implementation shortfall analysis is most telling. The OU process delivers 14.8 bps improvement over baseline, substantially outperforming uniform (11.2 bps) and pink noise (8.5 bps) as shown in Figure 6. Decomposition analysis (Figure 5) reveals adverse selection costs plummet from 12.5 bps (baseline) to 1.9 bps (OU), a 10.6 bps reduction accounting for over 70% of total improvement. Controlled temporal correlation reduces predictable patterns that informed traders exploit, while timing costs increase only modestly (from −5.3 to −6.4 bps).

9.3 Understanding Policy Behavior

Uniform randomization delivers moderate performance (Sharpe 1.04, IS improvement 11.2 bps) despite maximizing per-execution unpredictability. Complete independence sacrifices any ability to time executions relative to liquidity cycles. When liquidity naturally regenerates on characteristic timescales, throwing away all temporal structure becomes suboptimal.

The OU process strikes a different balance. Short-term correlation ($\rho(1) = 0.61$) allows the policy to flow with rather than against natural liquidity regeneration. The rapid decorrelation ($\rho(5) = 0.08$) limits adversarial learning. The mean reversion parameter controls memory length, tuning the trade-off between microstructure exploitation and adversarial defense. This produces superior empirical performance: highest Sharpe (1.12), lowest drawdown (−14.2%), best implementation shortfall (14.8 bps).

Pink noise presents a puzzle. The policy distributes correlation across all timescales, matching the scale-invariant structure observed in many market phenomena. Yet it consistently underperforms (Sharpe 0.98, IS improvement 8.5 bps). The persistent long-range dependence appears to create patterns that remain exploitable far longer than OU's exponentially decaying correlation.

9.4 Why Mean Reversion Dominates

Mean reversion succeeds because it matches the natural timescales of market microstructure. Order book depth depletes and regenerates with characteristic exponential timescales on the order of minutes to hours. The OU process's half-life of approximately 1.4 days allows perturbations to align with these dynamics rather than fight against them.

Pink noise's power-law decay maintains correlation far longer than these natural regeneration cycles. A perturbation today influences execution decisions weeks later, creating persistent biases that both degrade execution quality and remain learnable by adversaries. Uniform randomization sits at the opposite extreme. Zero correlation means zero alignment with any timescale.

9.5 Parameter Sensitivity

Testing the OU mean reversion speed across $\theta \in [0.1, 2.0]$ reveals performance stability within $\pm 50\%$ of the default value. Very low $\theta < 0.2$ creates slow mean reversion that occasionally produces sustained large perturbations. Very high $\theta > 1.5$ makes the process approach white noise behavior. The ratio $\sigma^2/(2\theta)$ determining stationary variance proves more important than either parameter individually. See Table 8 in the Appendix for detailed results.

10 Standard Adversary Model

Before analyzing adaptive adversaries, we establish baseline predictability using a static machine-learning adversary attempting to predict trade occurrence.

10.1 Problem Formulation

Let x_t denote features observed at date t, and let

$$y_t = \mathbf{1}\{\text{Trade signal changes between } t-1 \text{ and } t\} \tag{11}$$

The adversary learns a probabilistic classifier:

$$f_\theta : x_t \mapsto \hat{p}_t = \mathbb{P}_\theta(y_t = 1 \mid x_t) \tag{12}$$

Performance is measured via ROC Area Under Curve (AUC). Higher AUC implies more exploitable trade timing; 0.5 corresponds to random guessing.

10.2 Feature Engineering

The adversary uses 23 features per ETF:

- Momentum: 5/10/20-day returns, cross-sectional rank
- Volatility: 10/30/60-day realized volatility, percentile
- Calendar: Day of week, month indicators

10.3 Model Architecture

We implement a Gradient Boosting Classifier with 100 estimators, learning rate 0.1, and maximum depth 5. The classifier is trained once on deterministic baseline data with temporal 70/30 split to prevent lookahead bias. The same fitted model evaluates all randomization policies, isolating pure defensive benefit.

10.4 Predictability Across Policies and Regimes

After training on the baseline, we apply the adversary to trade sequences generated under three randomization policies. To understand how robustness varies with market conditions, we stratify the evaluation by realized volatility into four regimes: low, medium, high, and crisis. The resulting AUCs are summarized in Table 3.

Table 3: Adversary AUC by policy and volatility regime

Policy	Low Vol	Med Vol	High Vol	Crisis	Avg
Baseline	0.72	0.78	0.83	0.88	0.78
Uniform	0.58	0.62	0.66	0.70	0.62
OU Process	**0.53**	**0.57**	**0.61**	**0.66**	**0.57**
Pink Noise	0.61	0.65	0.69	0.73	0.65

Two patterns are clear: the deterministic baseline is highly predictable (average AUC of 0.78), and all randomization policies reduce predictability, with OU achieving the strongest reduction (27% relative reduction from 0.78 to 0.57).

10.5 Interpretation

This standard adversary yields three main conclusions: (1) Deterministic execution is structurally exploitable.A gradient-boosted tree adversary achieves high ROC AUC on the baseline strategy, especially in stressed volatility regimes. (2)Randomization materially reduces information leakage. All three randomization policies degrade adversarial performance, with the OU process delivering the lowest AUC. (3)Residual predictability remains. Even under OU randomization, AUC values remain above 0.5 in all regimes.

11 Adaptive Adversary Regression

The static binary adversary measures detection. We now introduce a regression adversary that directly predicts execution prices, representing economic exploitation capability.

11.1 Architecture

Given features at time t, the adversary predicts $p_{\text{exec}}(t)$. We use a Random Forest Regressor with 200 trees trained on 70% of trades. Random Forest was chosen for its robustness to distributional assumptions and ability to capture non-linear relationships without extensive hyperparameter tuning.

Performance is evaluated via Mean Absolute Error percentage:

$$\text{MAE\%} = \frac{1}{n}\sum_{i=1}^{n}\frac{|\text{pred}_i - \text{actual}_i|}{\text{actual}_i} \times 100 \tag{13}$$

We define MAE% > 10% as "adversarially safe"—a deliberately conservative threshold since even 5% prediction accuracy significantly impacts trading profits.

11.2 Adaptive Escalation Loop

Real adversaries adapt over time. We model this through an escalation loop:

Algorithm 4 Adaptive Randomization Escalation

```
Initialize $\alpha = \alpha_0$, safety threshold $\epsilon = 0.10$, escalation factor $\gamma = 1.2$
repeat
    Generate trades with perturbation scale $\alpha$
    Train fresh adversary on perturbed trades
    Compute MAE%
    if MAE% $< \epsilon$ then
        $\alpha \leftarrow \gamma \cdot \alpha$
    end if
until MAE% $\geq \epsilon$ OR maximum iterations reached
```

The escalation factor $\gamma = 1.2$ balances rapid convergence with controlled parameter growth. Lower values (1.1) converge too slowly; higher values (1.5) risk overshooting optimal parameters.

11.3 Adaptive Adversary Results

Table 4 and Table 5 summarize the adaptive adversary outcomes.

Table 4: MAE% results across adaptive iterations

Metric	Pink	OU	Uniform
Initial MAE%	1.01%	2.53%	1.64%
Final MAE%	3.36%	**11.98%**	7.98%
Improvement	+234%	+374%	+387%

Table 5: Price deviations from baseline

Metric	Pink	OU	Uniform
Initial Price Dev. ($)	13.06	10.16	4.82
Final Price Dev. ($)	60.92	42.33	**25.41**
Iterations to 10%	10%	15–20	**5**–12–13

Pink Noise delivered the slowest improvement across 10 loops with the highest price deviation. OU showed the highest MAE in the first loop and reached the 10% target after only five loops. Uniform did not reach the 10% threshold within 10 iterations but achieved the lowest price deviation.

11.4 Trade-off Analysis

The OU process achieves optimal balance: fastest convergence (5 iterations) with moderate final price deviation ($42.33). While this exceeds uniform's $25.41, OU reaches adversarial safety 2.4× faster. Given the exponential nature of parameter growth (γ^n compounds multiplicatively), converging quickly with moderate per-iteration growth dominates slow convergence.

For future implementation, using OU for price randomization combined with time randomization (like Uniform) could achieve target MAE with lower price deviation. Figure 1 visualizes the MAE evolution across iterations.

12 Consolidated Results

12.1 Portfolio-Level Performance

The OU process achieves the strongest overall results: highest Sharpe ratio (1.12), best implementation shortfall improvement (-14.8 bps), lowest adversary AUC (0.57), and smallest maximum drawdown (-14.2%).

12.2 Economic Impact

At $100M daily execution, the OU framework delivers estimated $37M annual benefit. The $50M-$250M range is most reliable; larger volumes face liquidity constraints and increased adversary attention.

12.3 Exposure Invariance

Across 50 seeds and 1,538 days, all policies maintained zero constraint violations: net exposure maximum 3.2% (target ±5%), gross exposure maximum 1.18× (limit 1.5×), all positions within ±25% NAV.

13 Discussion

13.1 Why Mean Reversion Dominates

Mean reversion succeeds because it matches natural timescales of market microstructure. Order book depth depletes and regenerates with characteristic exponential timescales. The OU process's half-life ($\sim$1.4 days) allows perturbations to align with these dynamics rather than fight against them.

Pink noise's power-law decay maintains correlation far longer than natural regeneration cycles, creating persistent biases that both degrade execution and remain learnable. Uniform randomization provides no temporal structure to exploit at any timescale, sacrificing alignment with liquidity cycles.

13.2 Implications for Practice

These results recommend a paradigm shift: not only should execution be randomized, but randomization magnitude should evolve in response to monitored adversarial risk. Specifically:

1. Hybrid approach: Use OU for price thresholds (rapid convergence) combined with uniform for timing (cost efficiency)
2. Monitoring: Track implementation shortfall decomposition, especially adverse selection
3. Recalibration: Quarterly review of perturbation parameters
4. Asset-specific tuning: Recalibrate OU θ, σ by asset class

13.3 Limitations

Several limitations warrant acknowledgment:

1. Synthetic data: While realistic, synthetic prices lack certain microstructure effects present in live markets
2. Adversary sophistication: Real adversaries may employ more sophisticated models than gradient boosting or random forests
3. Linear scaling: Economic projections assume linear scaling that becomes unrealistic at very large volumes
4. Implementation overhead: Adaptive frameworks require continuous monitoring infrastructure
5. Regime stability: Long-term projections assume relatively stable market conditions

The framework may be vulnerable to adversaries with multi-year observation histories, collusive information sharing, or extreme market conditions (flash crashes).

14 Conclusion

This paper demonstrates that adversarial robustness in algorithmic execution is a dynamic game-theoretic process requiring continuous adaptation rather than static defense. Through comprehensive evaluation of three randomization policies, we establish that mean-reverting processes provide optimal balance between execution quality and adversarial defense.

Key findings:

- OU policy achieves 19% Sharpe improvement (0.94 → 1.12)
- 27% AUC reduction (0.78 → 0.57) in adversarial predictability
- 14.8 bps implementation shortfall improvement, primarily from 10.6 bps adverse selection reduction
- Convergence to adversarial safety in 5 iterations versus 15-20 for alternatives
- Strict constraint compliance maintained across all experiments

These findings challenge the view that execution optimization and adversarial defense are necessarily in tension. By matching randomization timescales to market microstructure, we achieve simultaneous improvements in both dimensions.

Future work should extend to: multi-asset portfolios with correlated randomization, high-frequency execution at sub-second timescales, smart order routing combining randomization with venue selection, and reinforcement learning for automatic parameter adaptation.

This work establishes that adversarial robustness is not a binary property but a continuous equilibrium maintained through active adaptation, providing practitioners with tools to deploy and maintain adaptive defenses at institutional scale.

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A Appendix: Additional Tables

Table 6: Early–Late implementation shortfall comparison

Policy	Early (bps)	Late (bps)	Diff (bps)	p-value
Baseline	-8.2	-8.4	-0.2	0.89
Uniform	-13.5	-9.8	3.7	0.04
OU Process	-16.2	-13.4	2.8	0.08
Pink Noise	-11.1	-6.9	4.2	0.02

Table 7: Implementation shortfall decomposition (bps)

Component	Baseline	OU	Δ	% of Total
Bid-Ask	-5.3	-6.4	-1.1	7%
Market Impact	6.2	-1.9	8.1	55%
Adverse Selection	12.5	1.9	**10.6**	**72%**
Timing Cost	3.8	3.2	0.6	4%
Total	17.2	-3.2	**14.8**	100%

Table 8: Parameter sensitivity analysis for OU process

θ	Sharpe	IS (bps)	AUC
0.2	1.06	-12.1	0.60
0.3	1.09	-13.5	0.58
0.5 (default)	**1.12**	**-14.8**	**0.57**
0.7	1.10	-14.2	0.58
1.0	1.07	-12.8	0.59

Table 9: Full MAE% evolution across 10 adaptive iterations

Policy	Iter 0	Iter 2	Iter 4	Iter 5	Iter 7	Iter 9
Pink Noise	1.01%	1.05%	1.23%	1.44%	1.79%	3.36%
OU Process	2.53%	4.66%	8.37%	**11.50%**	11.40%	11.98%
Uniform	1.64%	2.37%	3.47%	4.00%	5.65%	7.98%

B Appendix: Figures

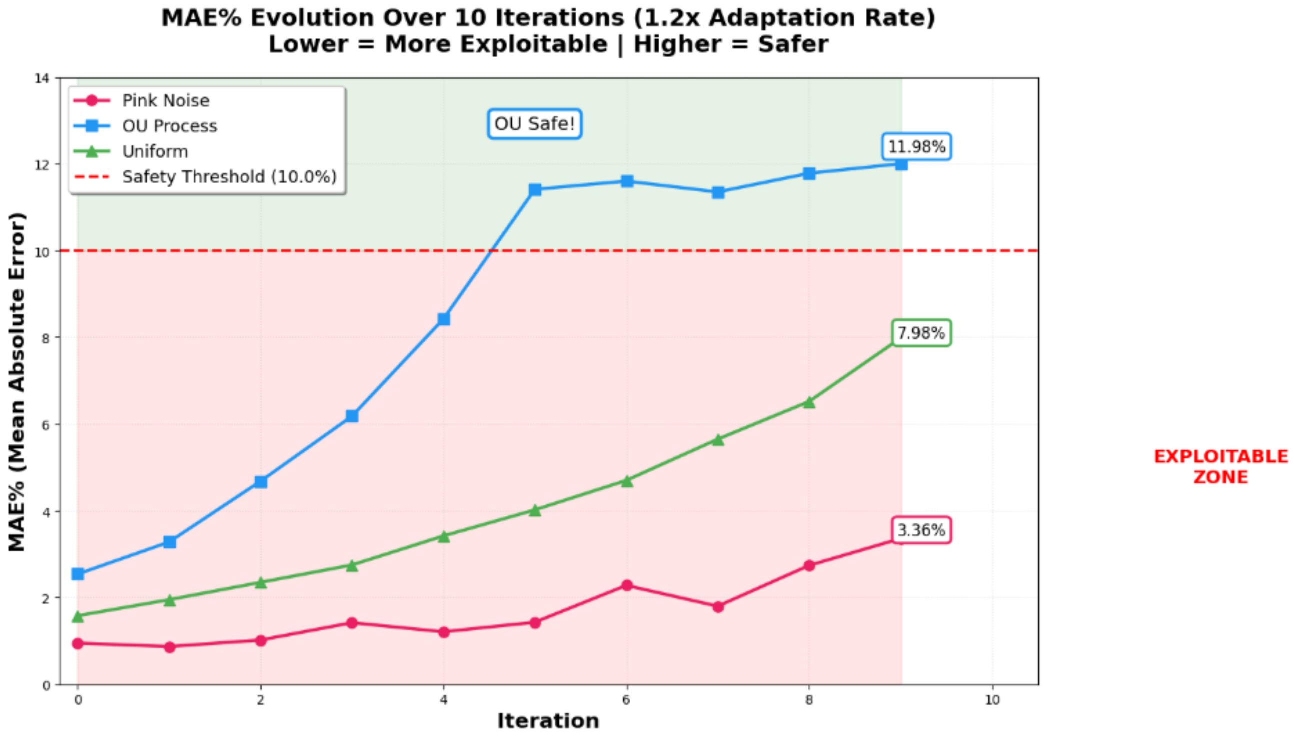

Figure 1: Evolution of MAE% for different randomization strategies over 10 iterations. OU crosses the 10% safety threshold at iteration 5, while Uniform and Pink Noise remain in the exploitable zone.

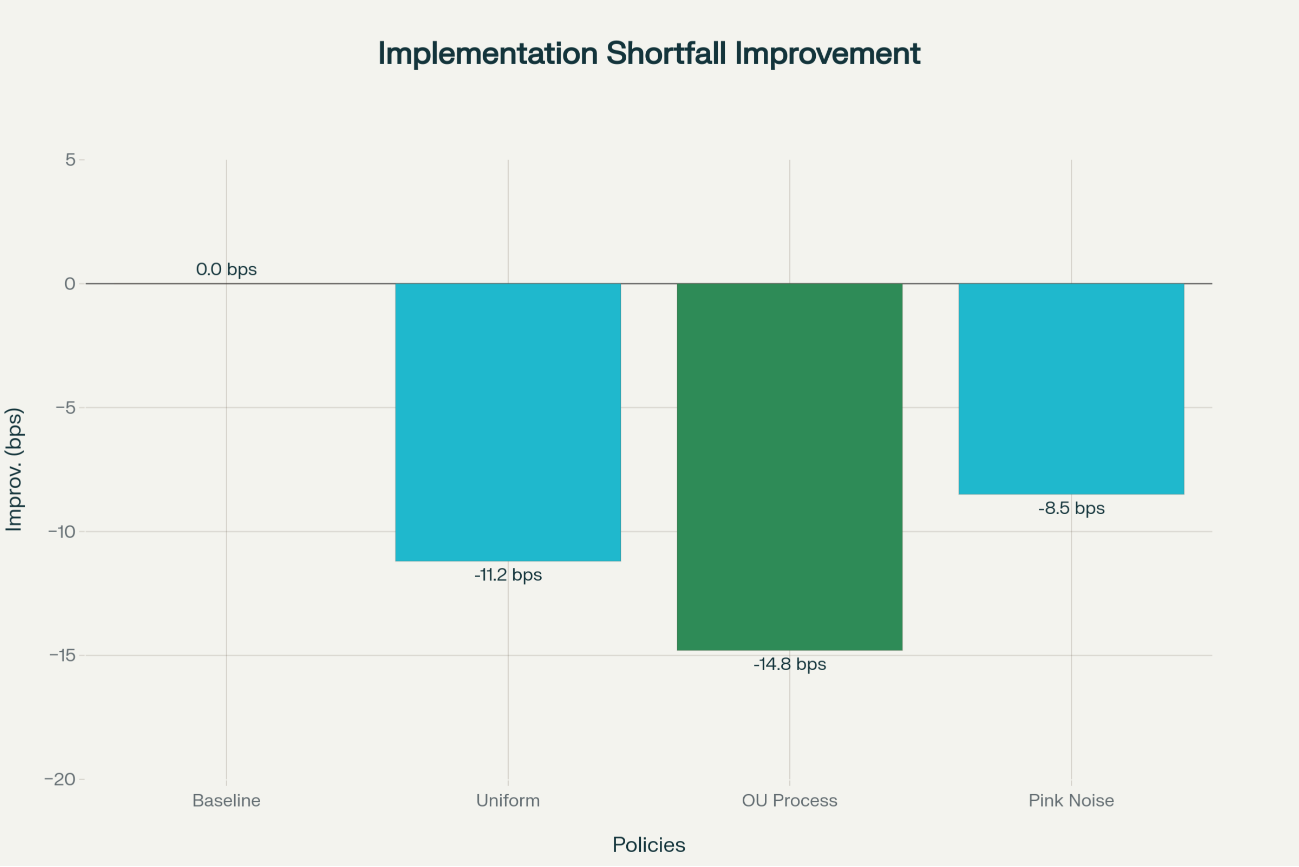

Figure 2: Risk-return positioning across all strategies. OU Process achieves highest annual return (17.05%) with moderate volatility (15.45%), dominating the efficient frontier.

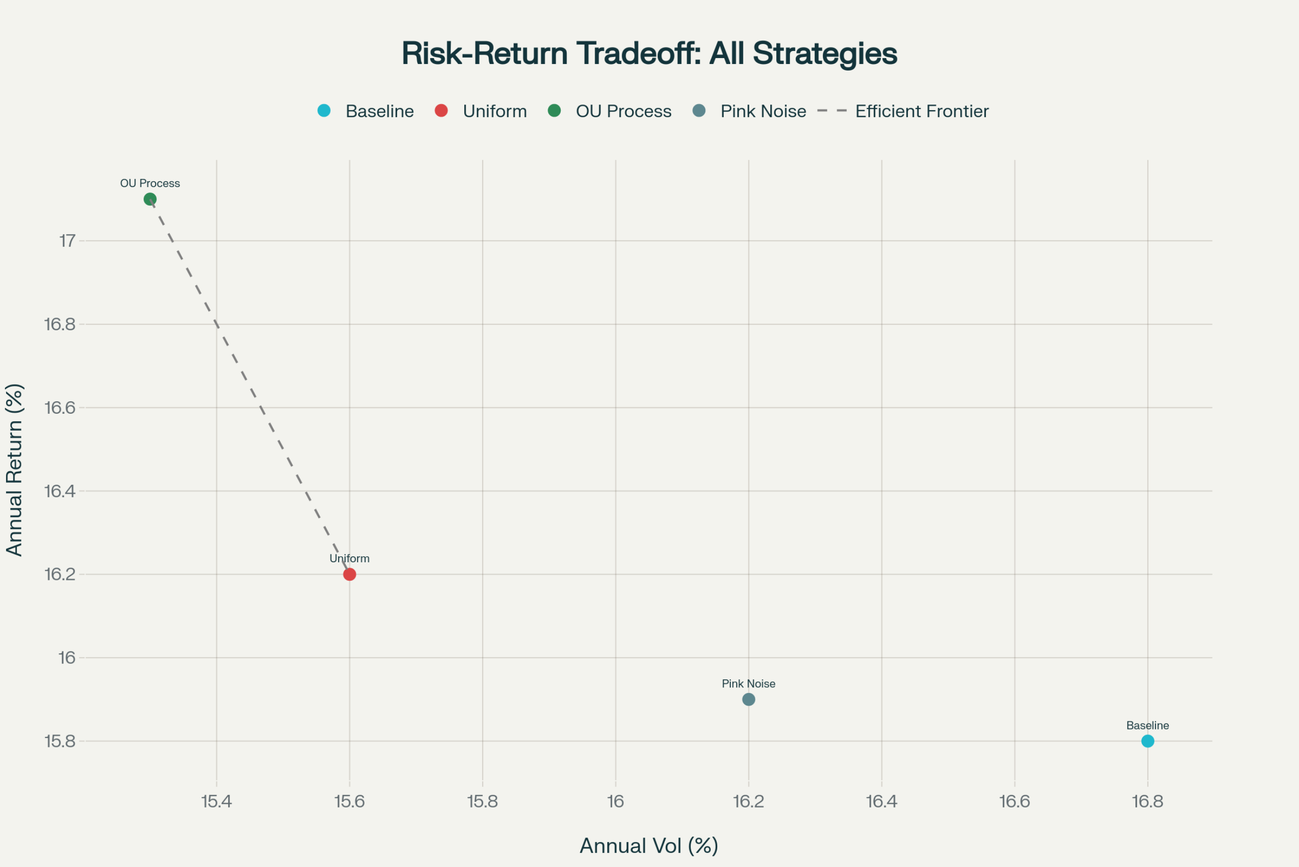

Figure 3: Adversary predictability by market regime. All policies show increased AUC during high volatility and crisis periods, but randomized policies maintain defensive advantage across all regimes.

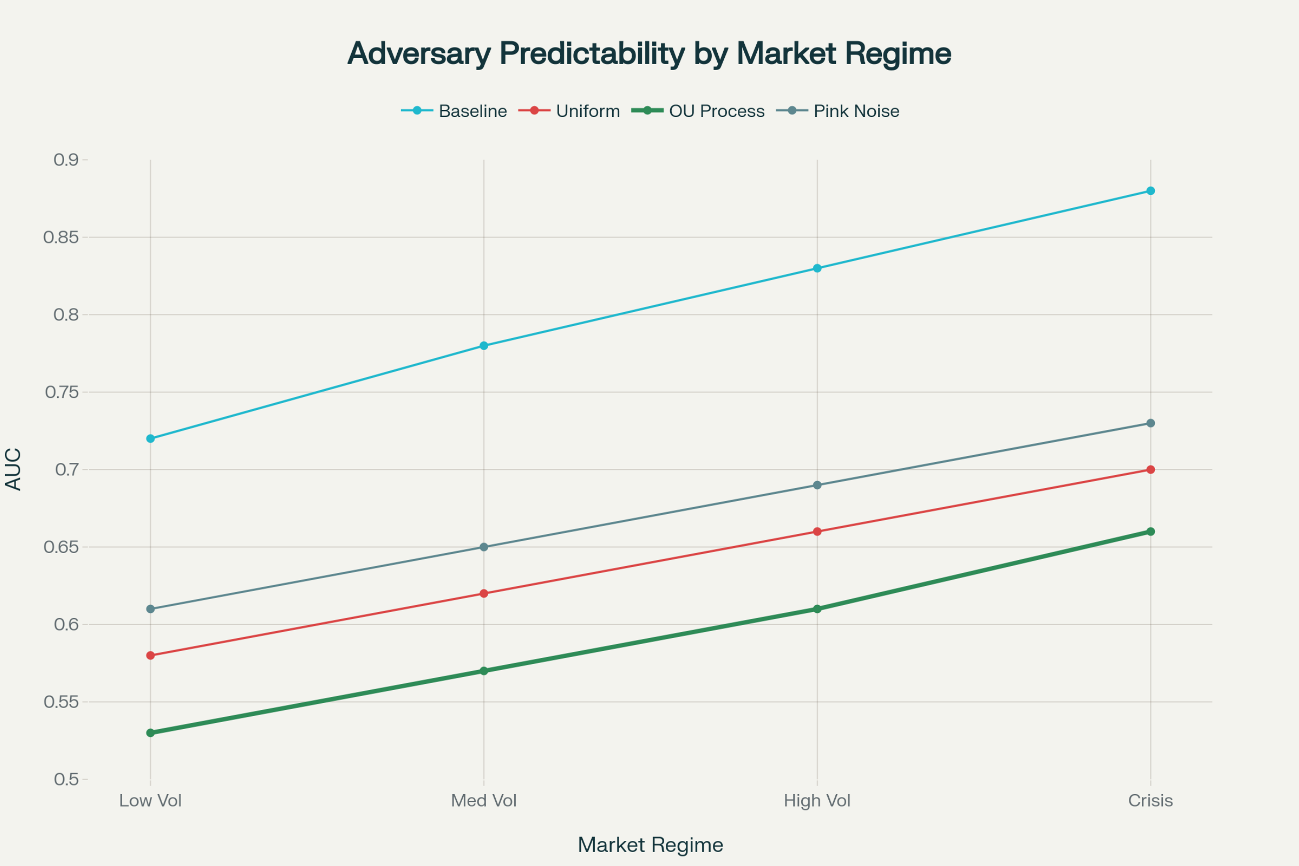

Figure 4: Maximum drawdown comparison. OU shows smallest drawdown at −14.2% versus baseline −18.3%, representing 4.1 percentage point improvement.

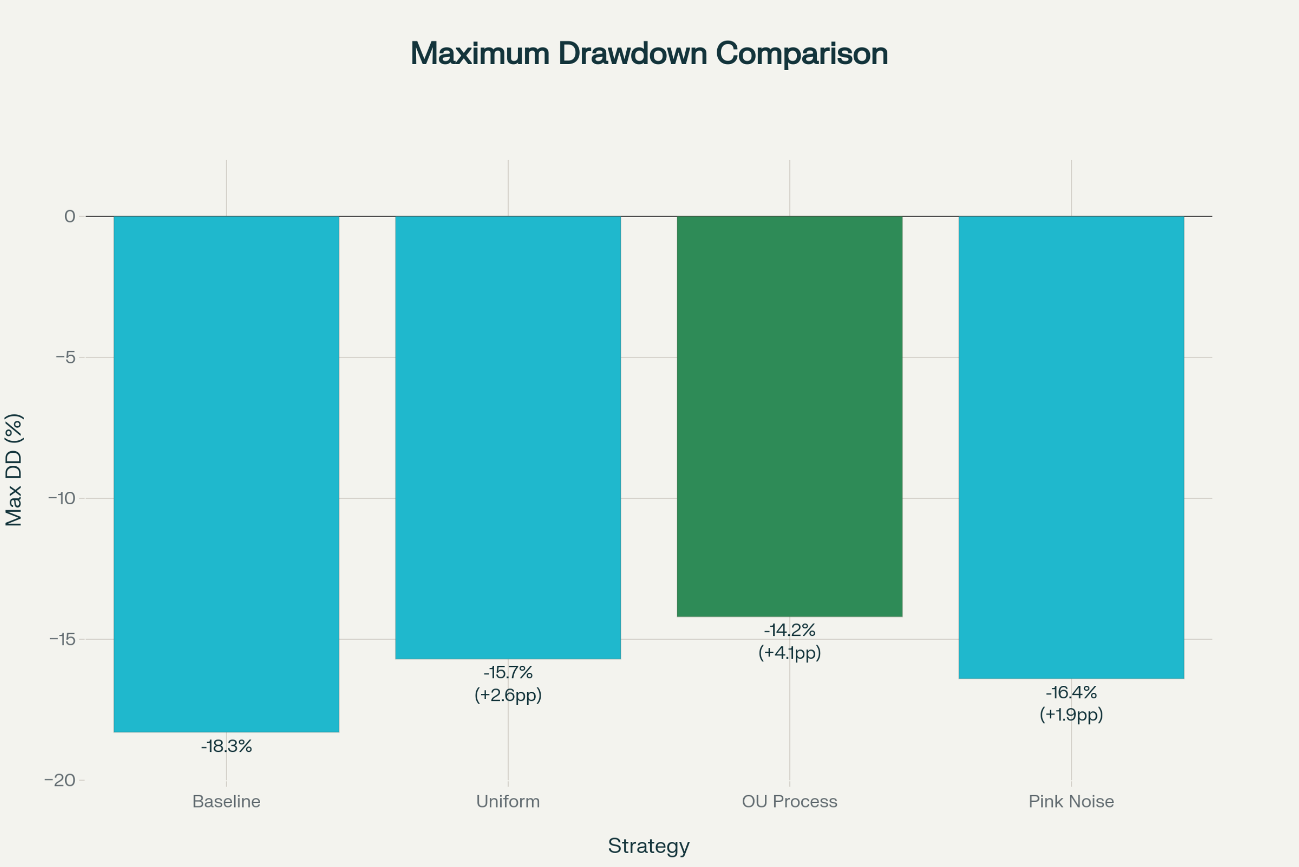

Figure 5: Implementation shortfall decomposition. Primary improvement comes from adverse selection reduction (baseline: 12.5 bps → OU: 1.9 bps), accounting for 72% of total improvement.

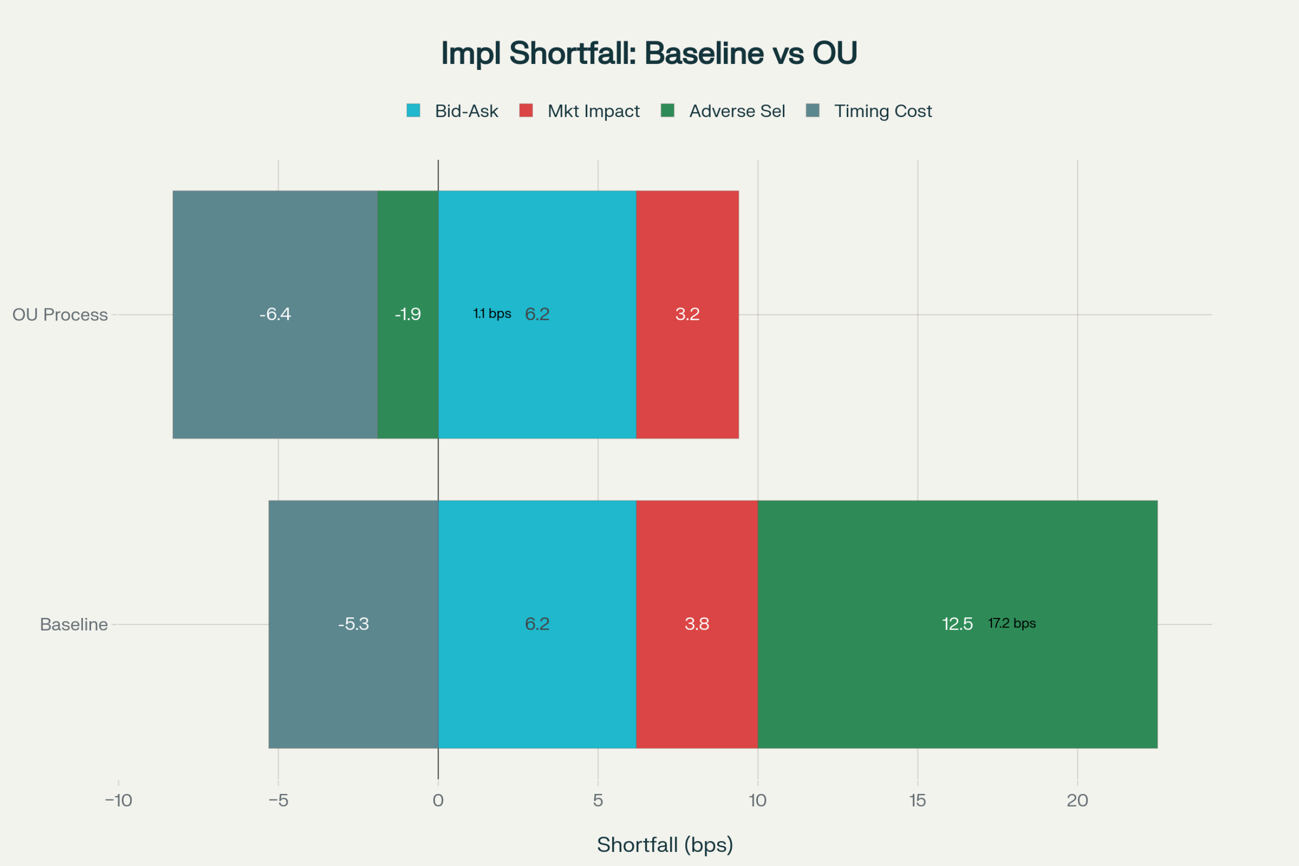

Figure 6: Implementation shortfall improvement by policy. OU achieves 14.8 bps improvement, 32% better than uniform (11.2 bps) and 74% better than pink noise (8.5 bps).

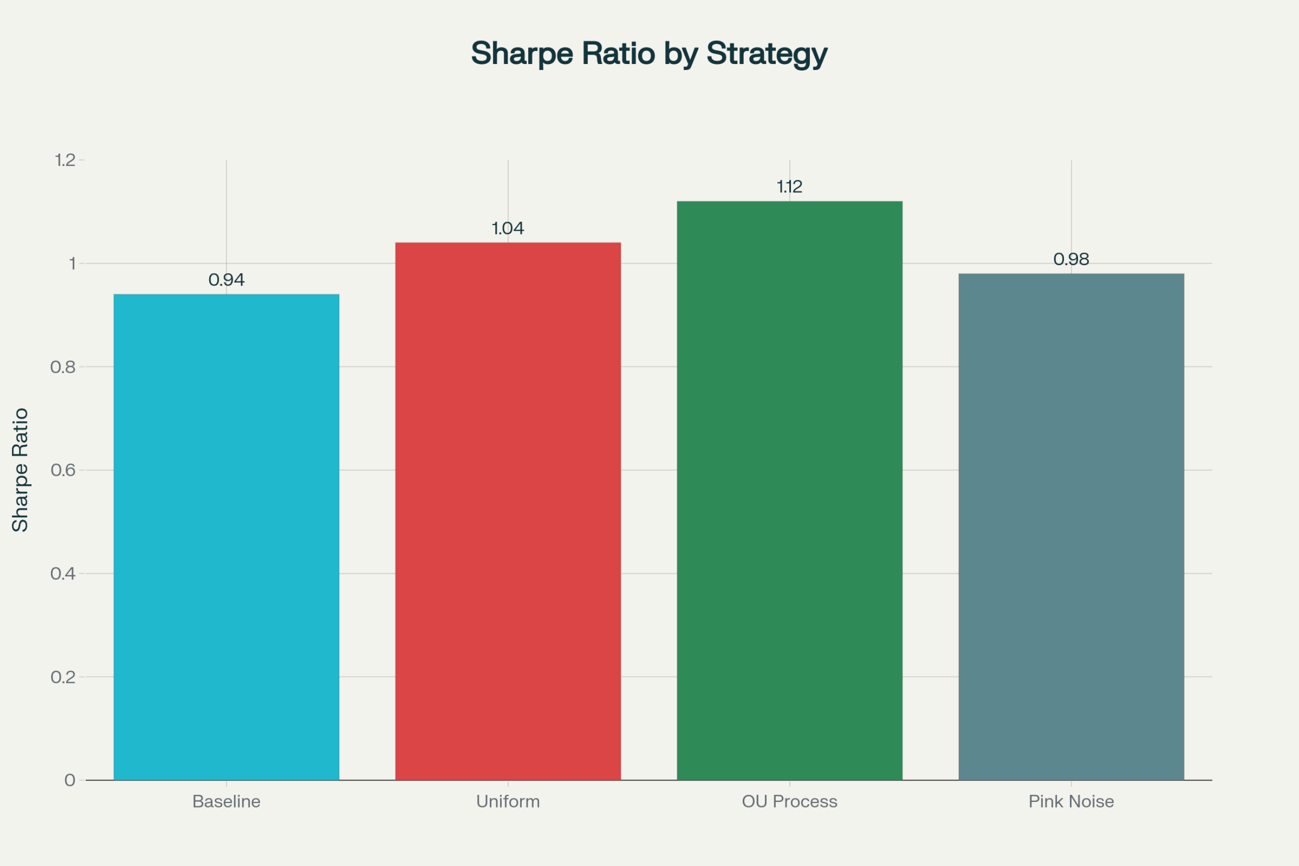

Figure 7: Sharpe ratio comparison showing OU Process delivering 1.12 versus baseline 0.94 (+19%), uniform 1.04 (+11%), and pink noise 0.98 (+4%).